Algebra 2

Glencoe

Name: _____

Period: _____

Chapter 9: Rational Expressions & Equations

Section	Lesson Objectives:	Assignment Problems	Date Due	Score
Prepare	Check prerequisite skills	*Homework*: Worksheet +&- Fractions	Feb 12	
9.1	Be able to simplify: 1. **Rational Expressions** 2. Complex Fractions	*Homework*: 15-43 odd, 48, 50, 51	Feb 16	
9.2	1. Determine the LCM of polynomials 2. Add & Subtract Rational Expressions	*Homework*: 15-41all , 52, 53	Feb 17	
9.3 A	Graph Rational Functions including the **Asymptotes & Point Discontinuities**	*Homework*: Study Guide and Intervention Worksheet pages 529 = 1-9 all and page 530 = 1-6 all	Feb 18	
9.3 B	9.3 continued …	*Homework*: Skills Practice Worksheet page 531 = 1-12 all & page 532=1-11	Feb 19	
QUIZ	**Mid-Chapter 9.1-9.3**	**Mid-Chapter QUIZ: 9.1-9.3**	**Feb 19**	
9.5	Be able to identify both graphs and equations according to their function type	*Classwork:* Worksheet p 545= do all *Homework*: Worksheet p 541= 1-9 , p 542 = 1-9, p 543 = 1-9, p 544 1-11	Feb 24	
9.6 A	Be able to solve **Rational Equations & Inequalities**	*Homework*: 1. Worksheet page 547 = 1-8 all 2. Pg 510 (book) = 11,13,17,23-29 odd	Feb 25	
9.6 B		*Homework*: 1. Worksheet page 548 = 1-6 all 2. Page 510 (book) = 15, 19, 21	Feb 26	
Test	**9.1-9.6**	**Partner Pre-Test on 9.1-9.6**	**March 1**	
Review	9.1-9.6	*Homework*: Study Guide and Review p. 513=1-6 all, 7-37 odd &Worksheet p 550 = 1-28 all	March 1	
TEST	**9.1-9.6**	**TEST on 9.1-9.6**	**March 3**	

Pre-Algebra

Name_____ ID: 1

9 Pre - Assignment

Date_____ Period____

Evaluate each expression. Leave all answers in fractions!

1) $2+\left(-\frac{11}{8}\right)$

2) $\left(-\frac{1}{3}\right)-\frac{1}{2}$

3) $\frac{3}{2}-\frac{1}{3}$

4) $\left(-\frac{3}{2}\right)+\left(-\frac{2}{5}\right)$

5) $\left(-\frac{3}{5}\right)-\frac{5}{7}$

6) $\left(-\frac{3}{4}\right)-\left(-\frac{9}{7}\right)$

Find the LCM of each.

7) 32, 40

8) 24, 40

9) $18y^2, 15xy^3$

10) $18v^3u, 27uv$

Evaluate each expression.

11) $\left(-2\frac{1}{6}\right)-\frac{4}{3}$

12) $\frac{3}{2}-\frac{13}{7}$

13) $2-\frac{1}{4}$

14) $\left(-\frac{2}{3}\right)-\frac{2}{7}$

Simplify each expression.

15) $\left(1\frac{2}{3}x^4+4\frac{1}{2}x\right)-\left(2x-\frac{1}{3}x^4\right)$

16) $\left(2x^4+1\frac{4}{7}\right)+\left(\frac{1}{8}x^4-3\frac{1}{4}\right)$

NAME _____ DATE _____ PERIOD ____

9-3 Study Guide and Intervention

Graphing Rational Functions

Vertical Asymptotes and Point Discontinuity

Rational Function	an equation of the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial expressions and $q(x) \neq 0$
Vertical Asymptote of the Graph of a Rational Function	An asymptote is a line that the graph of a function approaches, but never crosses. If the simplified form of the related rational expression is undefined for $x = a$, then $x = a$ is a vertical asymptote.
Point Discontinuity of the Graph of a Rational Function	Point discontinuity is like a hole in a graph. If the original related expression is undefined for $x = a$ but the simplified expression is defined for $x = a$, then there is a hole in the graph at $x = a$.

Example **Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x) = \frac{4x^2 + x - 3}{x^2 - 1}$.**

First factor the numerator and the denominator of the rational expression.

$$f(x) = \frac{4x^2 + x - 3}{x^2 - 1} = \frac{(4x - 3)(x + 1)}{(x + 1)(x - 1)}$$

The function is undefined for $x = 1$ and $x = -1$.

Since $\frac{(4x - 3)(x + 1)}{(x + 1)(x - 1)} = \frac{4x - 3}{x - 1}$, $x = 1$ is a vertical asymptote. The simplified expression is defined for $x = -1$, so this value represents a hole in the graph.

Exercises

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \frac{4}{x^2 + 3x - 10}$

2. $f(x) = \frac{2x^2 - x - 10}{2x - 5}$

3. $f(x) = \frac{x^2 - x - 12}{x^2 - 4x}$

4. $f(x) = \frac{3x - 1}{3x^2 + 5x - 2}$

5. $f(x) = \frac{x^2 - 6x - 7}{x^2 + 6x - 7}$

6. $f(x) = \frac{3x^2 - 5x - 2}{x + 3}$

7. $f(x) = \frac{x + 1}{x^2 - 6x + 5}$

8. $f(x) = \frac{2x^2 - x - 3}{2x^2 + 3x - 9}$

9. $f(x) = \frac{x^3 - 2x^2 - 5x + 6}{x^2 - 4x + 3}$

Lesson 9-3

NAME _____ DATE _____ PERIOD ____

9-3 Study Guide and Intervention *(continued)*

Graphing Rational Functions

Graph Rational Functions Use the following steps to graph a rational function.

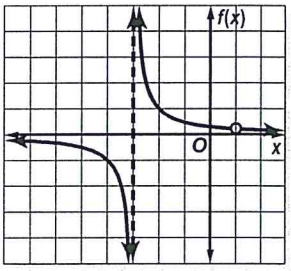
Step 1	First see if the function has any vertical asymptotes or point discontinuities.
Step 2	Draw any vertical asymptotes.
Step 3	Make a table of values.
Step 4	Plot the points and draw the graph.

Example **Graph** $f(x) = \dfrac{x - 1}{x^2 + 2x - 3}$.

$\dfrac{x - 1}{x^2 + 2x - 3} = \dfrac{x - 1}{(x - 1)(x + 3)}$ or $\dfrac{1}{x + 3}$

Therefore the graph of $f(x)$ has an asymptote at $x = -3$ and a point discontinuity at $x = 1$.

Make a table of values. Plot the points and draw the graph.

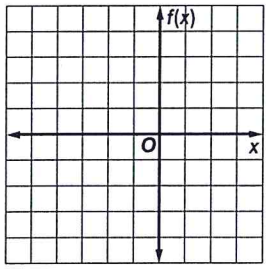


x	−2.5	−2	−1	−3.5	−4	−5
$f(x)$	2	1	0.5	−2	−1	−0.5

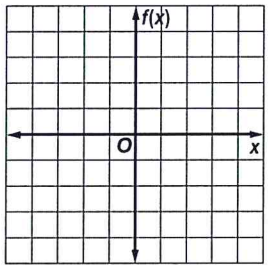
Exercises

Graph each rational function.

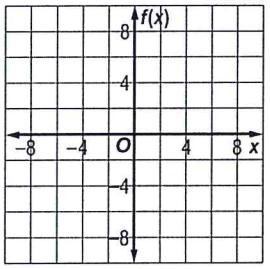
1. $f(x) = \dfrac{3}{x + 1}$



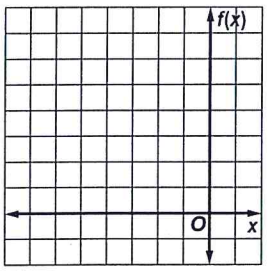
2. $f(x) = \dfrac{2}{x}$



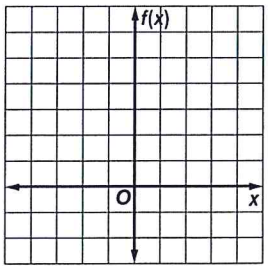
3. $f(x) = \dfrac{2x + 1}{x - 3}$



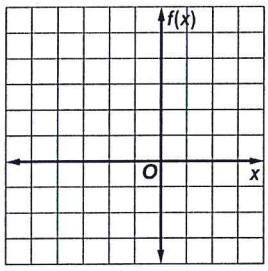
4. $f(x) = \dfrac{2}{(x + 3)^2}$



5. $f(x) = \dfrac{x^2 - x - 6}{x - 3}$



6. $f(x) = \dfrac{x^2 - 6x + 8}{x^2 - x - 2}$



NAME _____ DATE _____ PERIOD ____

9-3 Study Guide and Intervention

Graphing Rational Functions

Vertical Asymptotes and Point Discontinuity

Rational Function	an equation of the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial expressions and $q(x) \neq 0$
Vertical Asymptote of the Graph of a Rational Function	An asymptote is a line that the graph of a function approaches, but never crosses. If the simplified form of the related rational expression is undefined for $x = a$, then $x = a$ is a vertical asymptote.
Point Discontinuity of the Graph of a Rational Function	Point discontinuity is like a hole in a graph. If the original related expression is undefined for $x = a$ but the simplified expression is defined for $x = a$, then there is a hole in the graph at $x = a$.

Example **Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x) = \frac{4x^2 + x - 3}{x^2 - 1}$.**

First factor the numerator and the denominator of the rational expression.

$f(x) = \frac{4x^2 + x - 3}{x^2 - 1} = \frac{(4x - 3)(x + 1)}{(x + 1)(x - 1)}$

The function is undefined for $x = 1$ and $x = -1$.

Since $\frac{(4x - 3)(x + 1)}{(x + 1)(x - 1)} = \frac{4x - 3}{x - 1}$, $x = 1$ is a vertical asymptote. The simplified expression is defined for $x = -1$, so this value represents a hole in the graph.

Exercises

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \frac{4}{x^2 + 3x - 10}$
   asymptotes: $x = 2$, $x = -5$

2. $f(x) = \frac{2x^2 - x - 10}{2x - 5}$
   hole: $x = \frac{5}{2}$

3. $f(x) = \frac{x^2 - x - 12}{x^2 - 4x}$
   asymptote: $x = 0$; hole $x = 4$

4. $f(x) = \frac{3x - 1}{3x^2 + 5x - 2}$
   asymptote: $x = -2$; hole: $x = \frac{1}{3}$

5. $f(x) = \frac{x^2 - 6x - 7}{x^2 + 6x - 7}$
   asymptotes: $x = 1$, $x = -7$

6. $f(x) = \frac{3x^2 - 5x - 2}{x + 3}$
   asymptote: $x = -3$

7. $f(x) = \frac{x + 1}{x^2 - 6x + 5}$
   asymptotes: $x = 1$, $x = 5$

8. $f(x) = \frac{2x^2 - x - 3}{2x^2 + 3x - 9}$
   asymptote: $x = -3$; hole: $x = \frac{3}{2}$

9. $f(x) = \frac{x^3 - 2x^2 - 5x + 6}{x^2 - 4x + 3}$
   holes: $x = 1$, $x = 3$

NAME _____ DATE _____ PERIOD ____

9-3 Study Guide and Intervention *(continued)*

Graphing Rational Functions

Graph Rational Functions

Use the following steps to graph a rational function.

Step 1	First see if the function has any vertical asymptotes or point discontinuities.
Step 2	Draw any vertical asymptotes.
Step 3	Make a table of values.
Step 4	Plot the points and draw the graph.

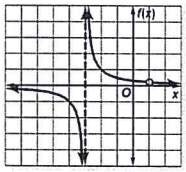
Example **Graph $f(x) = \frac{x - 1}{x^2 + 2x - 3}$.**

$\frac{x - 1}{x^2 + 2x - 3} = \frac{x - 1}{(x - 1)(x + 3)}$ or $\frac{1}{x + 3}$

Therefore the graph of $f(x)$ has an asymptote at $x = -3$ and a point discontinuity at $x = 1$.

Make a table of values. Plot the points and draw the graph.

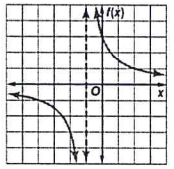
x	−2.5	−2	−1	−3.5	−4	−5
$f(x)$	2	1	0.5	−2	−1	−0.5



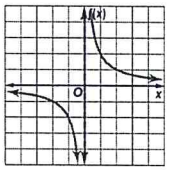
Exercises

Graph each rational function.

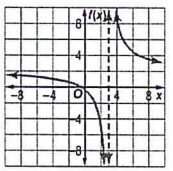
1. $f(x) = \frac{3}{x + 1}$



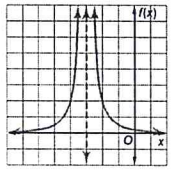
2. $f(x) = \frac{2}{x}$



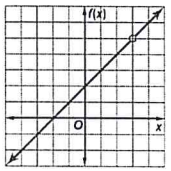
3. $f(x) = \frac{2x + 1}{x - 3}$



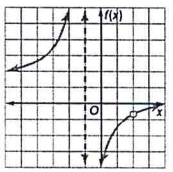
4. $f(x) = \frac{2}{(x + 3)^2}$



5. $f(x) = \frac{x^2 - x - 6}{x - 3}$



6. $f(x) = \frac{x^2 - 6x + 8}{x^2 - x - 2}$



NAME _____ DATE _____ PERIOD _____

9-3 Skills Practice

Graphing Rational Functions

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \dfrac{3}{x^2 - 2x - 8}$

2. $f(x) = \dfrac{10}{x^2 - 13x + 36}$

3. $f(x) = \dfrac{x + 12}{x^2 + 10x - 24}$

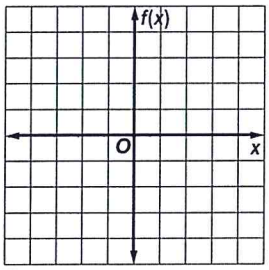
4. $f(x) = \dfrac{x - 1}{x^2 - 4x + 3}$

5. $f(x) = \dfrac{x^2 + 8x + 12}{x + 2}$

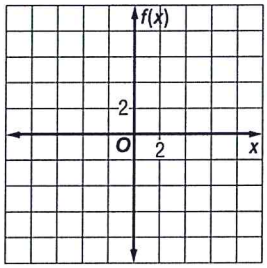
6. $f(x) = \dfrac{x^2 + x - 12}{x - 3}$

Graph each rational function.

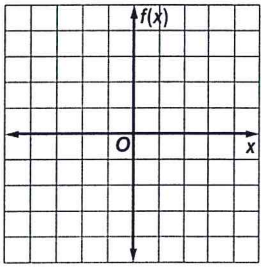
7. $f(x) = \dfrac{-3}{x}$



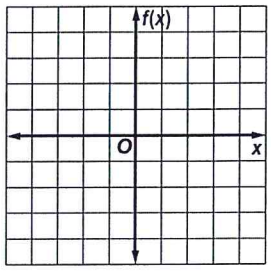
8. $f(x) = \dfrac{10}{x}$



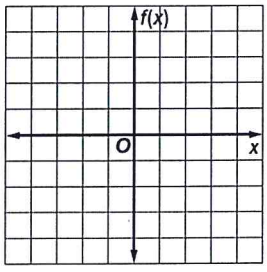
9. $f(x) = \dfrac{-4}{x}$



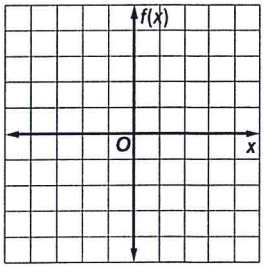
10. $f(x) = \dfrac{2}{x - 1}$



11. $f(x) = \dfrac{x}{x + 2}$



12. $f(x) = \dfrac{x^2 - 4}{x - 2}$



Lesson 9-3

NAME _____ DATE _____ PERIOD ____

9-3 Practice

Graphing Rational Functions

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \dfrac{6}{x^2 + 3x - 10}$

2. $f(x) = \dfrac{x - 7}{x^2 - 10x + 21}$

3. $f(x) = \dfrac{x - 2}{x^2 + 4x + 4}$

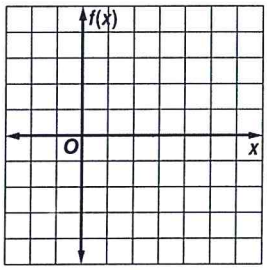
4. $f(x) = \dfrac{x^2 - 100}{x + 10}$

5. $f(x) = \dfrac{x^2 - 2x - 24}{x - 6}$

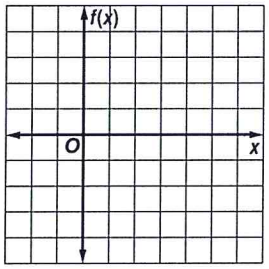
6. $f(x) = \dfrac{x^2 + 9x + 20}{x + 5}$

Graph each rational function.

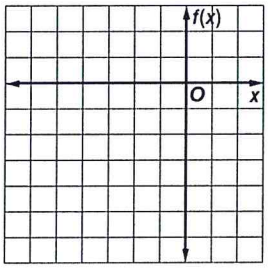
7. $f(x) = \dfrac{-4}{x - 2}$



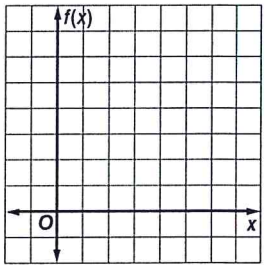
8. $f(x) = \dfrac{x - 3}{x - 2}$



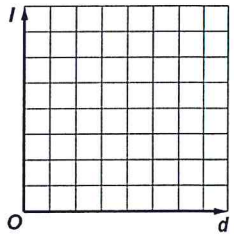
9. $f(x) = \dfrac{3x}{(x + 3)^2}$



10. PAINTING Working alone, Tawa can give the shed a coat of paint in 6 hours. It takes her father x hours working alone to give the shed a coat of paint. The equation $f(x) = \dfrac{6 + x}{6x}$ describes the portion of the job Tawa and her father working together can complete in 1 hour. Graph $f(x) = \dfrac{6 + x}{6x}$ for $x \geq 0, y \geq 0$. If Tawa's father can complete the job in 4 hours alone, what portion of the job can they complete together in 1 hour?



11. LIGHT The relationship between the illumination an object receives from a light source of I foot-candles and the square of the distance d in feet of the object from the source can be modeled by $I(d) = \dfrac{4500}{d^2}$. Graph the function $I(d) = \dfrac{4500}{d^2}$ for $0 \leq I \leq 80$ and $0 \leq d \leq 80$. What is the illumination in foot-candles that the object receives at a distance of 20 feet from the light source?



NAME _____ DATE _____ PERIOD ____

9-3 Skills Practice

Graphing Rational Functions

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \frac{3}{x^2 - 2x - 8}$
asymptotes: $x = 4$, $x = -2$

2. $f(x) = \frac{10}{x^2 - 13x + 36}$
asymptotes: $x = 4$, $x = 9$

3. $f(x) = \frac{x + 12}{x^2 + 10x - 24}$
asymptote: $x = 2$; hole: $x = -12$

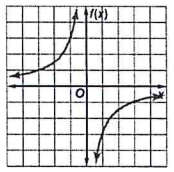
4. $f(x) = \frac{x - 1}{x^2 - 4x + 3}$
asymptote: $x = 3$; hole: $x = 1$

5. $f(x) = \frac{x^2 + 8x + 12}{x + 2}$
hole: $x = -2$

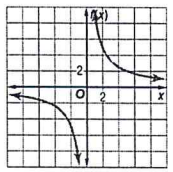
6. $f(x) = \frac{x^2 + x - 12}{x - 3}$
hole: $x = 3$

Graph each rational function.

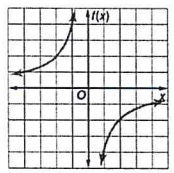
7. $f(x) = \frac{-3}{x}$



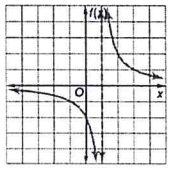
8. $f(x) = \frac{10}{x}$



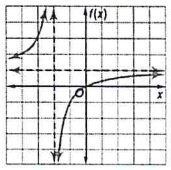
9. $f(x) = \frac{-4}{x}$



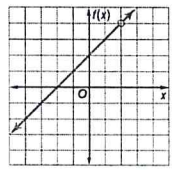
10. $f(x) = \frac{2}{x - 1}$



11. $f(x) = \frac{x}{x + 2}$



12. $f(x) = \frac{x^2 - 4}{x - 2}$



NAME _____ DATE _____ PERIOD ____

9-3 Practice (Average)

Graphing Rational Functions

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \frac{6}{x^2 + 3x - 10}$
asymptotes: $x = 2$, $x = -5$

2. $f(x) = \frac{x - 7}{x^2 - 10x + 21}$
asymptote: $x = 3$; hole: $x = 7$

3. $f(x) = \frac{x - 2}{x^2 + 4x + 4}$
asymptote: $x = -2$

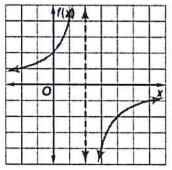
4. $f(x) = \frac{x^2 - 100}{x + 10}$
hole: $x = -10$

5. $f(x) = \frac{x^2 - 2x - 24}{x - 6}$
hole: $x = 6$

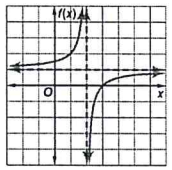
6. $f(x) = \frac{x^2 + 9x + 20}{x + 5}$
hole: $x = -5$

Graph each rational function.

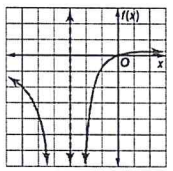
7. $f(x) = \frac{-4}{x - 2}$



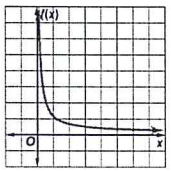
8. $f(x) = \frac{x - 3}{x - 2}$



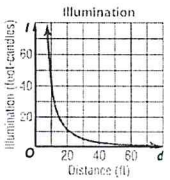
9. $f(x) = \frac{3x}{(x + 3)^2}$



10. **PAINTING** Working alone, Tawa can give the shed a coat of paint in 6 hours. It takes her father x hours working alone to give the shed a coat of paint. The equation $f(x) = \frac{6 + x}{6x}$ describes the portion of the job Tawa and her father working together can complete in 1 hour. Graph $f(x) = \frac{6 + x}{6x}$ for $x \geq 0, y \geq 0$. If Tawa's father can complete the job in 4 hours alone, what portion of the job can they complete together in 1 hour? $\frac{5}{12}$



11. **LIGHT** The relationship between the illumination an object receives from a light source of I foot-candles and the square of the distance d in feet of the object from the source can be modeled by $I(d) = \frac{4500}{d^2}$. Graph the function $I(d) = \frac{4500}{d^2}$ for $0 \leq I \leq 80$ and $0 \leq d \leq 80$. What is the illumination in foot-candles that the object receives at a distance of 20 feet from the light source? 11.25 foot-candles



Lesson 9-3

Answers

NAME _____ DATE _____ PERIOD _____

9-5 Reading to Learn Mathematics

Classes of Functions

Pre-Activity **How can graphs of functions be used to determine a person's weight on a different planet?**

Read the introduction to Lesson 9-5 at the top of page 499 in your textbook.

- Based on the graph, estimate the weight on Mars of a child who weighs 40 pounds on Earth.

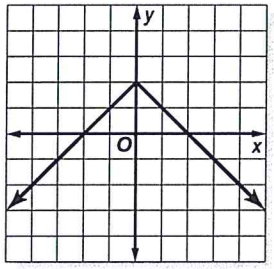
- Although the graph does not extend far enough to the right to read it directly from the graph, use the weight you found above and your knowledge that this graph represents direct variation to estimate the weight on Mars of a woman who weighs 120 pounds on Earth.

Reading the Lesson

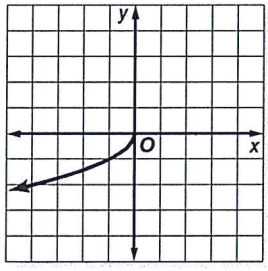
1. Match each graph below with the type of function it represents. Some types may be used more than once and others not at all.

I. square root **II.** quadratic **III.** absolute value **IV.** rational
V. greatest integer **VI.** constant **VII.** identity

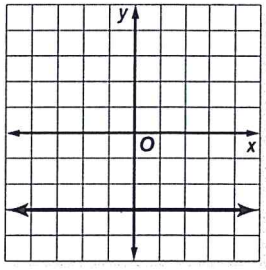
a.



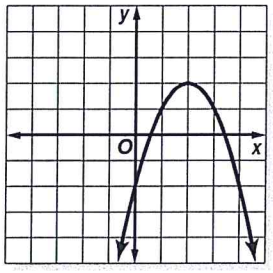
b.



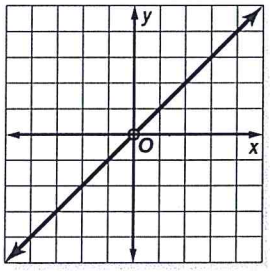
c.



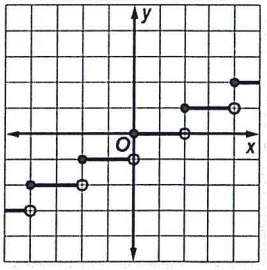
d.



e.



f.



Helping You Remember

2. How can the symbolic definition of absolute value that you learned in Lesson 1-4 help you to remember the graph of the function $f(x) = |x|$?

Lesson 9-5

NAME _____ DATE _____ PERIOD ____

9-5 Study Guide and Intervention

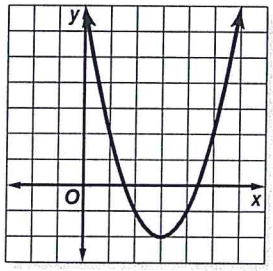
Classes of Functions

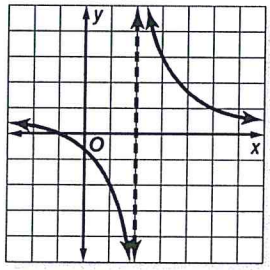
Identify Graphs You should be familiar with the graphs of the following functions.

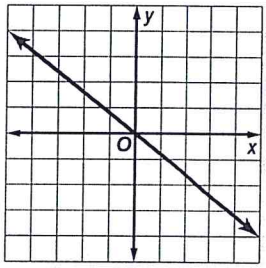
Function	Description of Graph
Constant	a horizontal line that crosses the y-axis at a
Direct Variation	a line that passes through the origin and is neither horizontal nor vertical
Identity	a line that passes through the point (a, a), where a is any real number
Greatest Integer	a step function
Absolute Value	V-shaped graph
Quadratic	a parabola
Square Root	a curve that starts at a point and curves in only one direction
Rational	a graph with one or more asymptotes and/or holes
Inverse Variation	a graph with 2 curved branches and 2 asymptotes, $x = 0$ and $y = 0$ (special case of rational function)

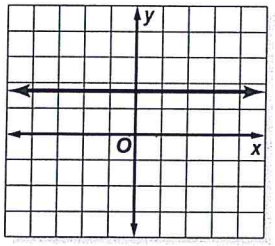
Exercises

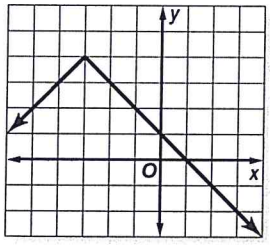
Identify the function represented by each graph.

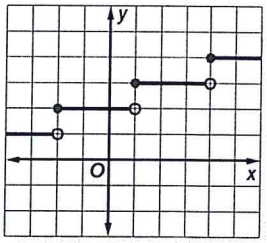
1. 

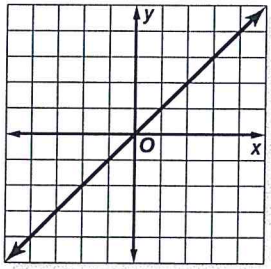
2. 

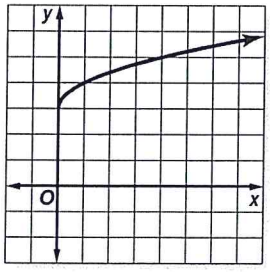
3. 

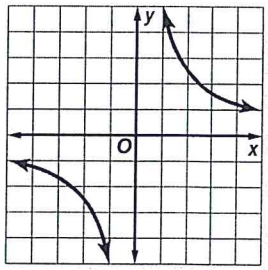
4. 

5. 

6. 

7. 

8. 

9. 

Lesson 9-5

NAME _____ DATE _____ PERIOD ____

9-5 Study Guide and Intervention *(continued)*

Classes of Functions

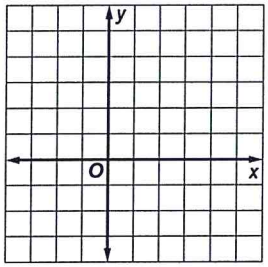
Identify Equations You should be able to graph the equations of the following functions.

Function	General Equation
Constant	$y = a$
Direct Variation	$y = ax$
Identity	$y = x$
Greatest Integer	equation includes a variable within the greatest integer symbol, $[\![\,]\!]$
Absolute Value	equation includes a variable within the absolute value symbol, $\lvert\,\rvert$
Quadratic	$y = ax^2 + bx + c$, where $a \neq 0$
Square Root	equation includes a variable beneath the radical sign, $\sqrt{\ }$
Rational	$y = \frac{p(x)}{q(x)}$
Inverse Variation	$y = \frac{a}{x}$

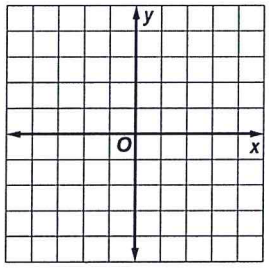
Exercises

Identify the function represented by each equation. Then graph the equation.

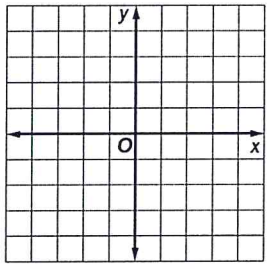
1. $y = \frac{6}{x}$



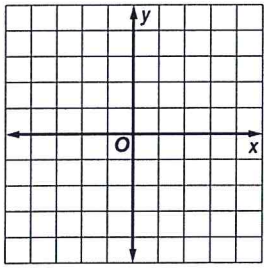
2. $y = \frac{4}{3}x$



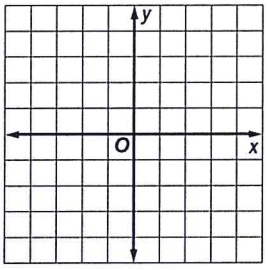
3. $y = -\frac{x^2}{2}$



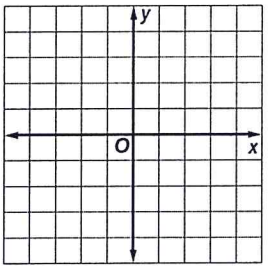
4. $y = |3x| - 1$



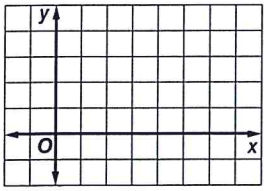
5. $y = -\frac{2}{x}$



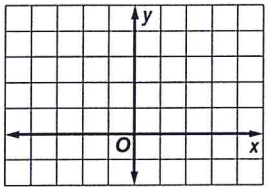
6. $y = \left[\!\left[\frac{x}{2}\right]\!\right]$



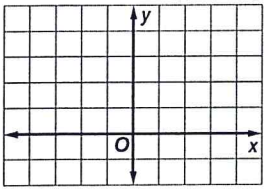
7. $y = \sqrt{x - 2}$



8. $y = 3.2$



9. $y = \frac{x^2 + 5x + 6}{x + 2}$



NAME _____ DATE _____ PERIOD ____

9-5 Study Guide and Intervention

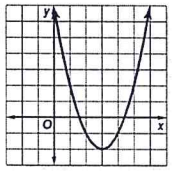
Classes of Functions

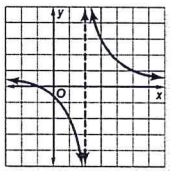
Identify Graphs You should be familiar with the graphs of the following functions.

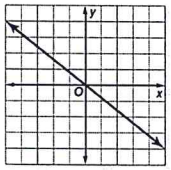
Function	Description of Graph
Constant	a horizontal line that crosses the y-axis at a
Direct Variation	a line that passes through the origin and is neither horizontal nor vertical
Identity	a line that passes through the point (a, a), where a is any real number
Greatest Integer	a step function
Absolute Value	V-shaped graph
Quadratic	a parabola
Square Root	a curve that starts at a point and curves in only one direction
Rational	a graph with one or more asymptotes and/or holes
Inverse Variation	a graph with 2 curved branches and 2 asymptotes, $x = 0$ and $y = 0$ (special case of rational function)

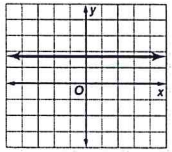
Exercises

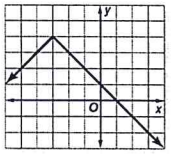
Identify the function represented by each graph.

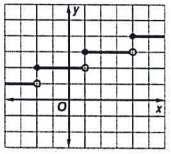
1.  quadratic

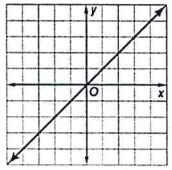
2.  rational

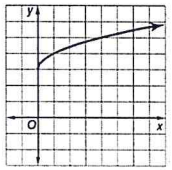
3.  direct variation

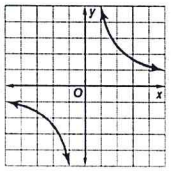
4.  constant

5.  absolute value

6.  greatest integer

7.  identity

8.  square root

9.  inverse variation

Lesson 9-5

NAME _____ DATE _____ PERIOD ____

9-5 Study Guide and Intervention *(continued)*

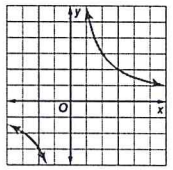
Classes of Functions

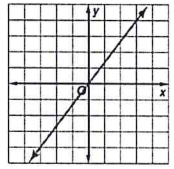
Identify Equations You should be able to graph the equations of the following functions.

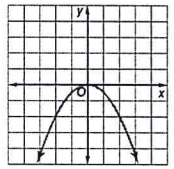
Function	General Equation
Constant	$y = a$
Direct Variation	$y = ax$
Identity	$y = x$
Greatest Integer	equation includes a variable within the greatest integer symbol, $[\![\ ]\!]$
Absolute Value	equation includes a variable within the absolute value symbol, $\lvert\ \rvert$
Quadratic	$y = ax^2 + bx + c$, where $a \neq 0$
Square Root	equation includes a variable beneath the radical sign, $\sqrt{\ }$
Rational	$y = \frac{p(x)}{q(x)}$
Inverse Variation	$y = \frac{a}{x}$

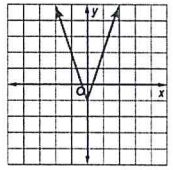
Exercises

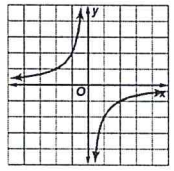
Identify the function represented by each equation. Then graph the equation.

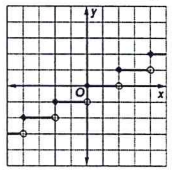
1. $y = \frac{6}{x}$ inverse variation 

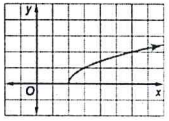
2. $y = \frac{4}{3}x$ direct variation 

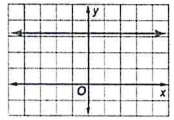
3. $y = -\frac{x^2}{2}$ quadratic 

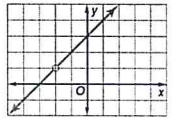
4. $y = |3x| - 1$ absolute value 

5. $y = -\frac{2}{x}$ inverse variation 

6. $y = \left[\!\left[\frac{x}{2}\right]\!\right]$ greatest integer 

7. $y = \sqrt{x - 2}$ square root 

8. $y = 3.2$ constant 

9. $y = \frac{x^2 + 5x + 6}{x + 2}$ rational 

NAME _____ DATE _____ PERIOD ____

9-6 Study Guide and Intervention

Solving Rational Equations and Inequalities

Solve Rational Equations A **rational equation** contains one or more rational expressions. To solve a rational equation, first multiply each side by the least common denominator of all of the denominators. Be sure to exclude any solution that would produce a denominator of zero.

Example **Solve** $\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$.

$\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$ Original equation

$10(x+1)\left(\frac{9}{10} + \frac{2}{x+1}\right) = 10(x+1)\left(\frac{2}{5}\right)$ Multiply each side by $10(x+1)$.

$9(x+1) + 2(10) = 4(x+1)$ Multiply.

$9x + 9 + 20 = 4x + 4$ Distributive Property

$5x = -25$ Subtract 4x and 29 from each side.

$x = -5$ Divide each side by 5.

Check

$\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$ Original equation

$\frac{9}{10} + \frac{2}{-5+1} \stackrel{?}{=} \frac{2}{5}$ $x = -5$

$\frac{9}{10} + \frac{2}{-4} \stackrel{?}{=} \frac{2}{5}$ Simplify.

$\frac{18}{20} - \frac{10}{20} \stackrel{?}{=} \frac{2}{5}$ Simplify.

$\frac{8}{20} \stackrel{?}{=} \frac{2}{5}$ Simplify.

$\frac{2}{5} = \frac{2}{5}$

Exercises

Solve each equation.

1. $\frac{2y}{3} - \frac{y+3}{6} = 2$

2. $\frac{4t-3}{5} - \frac{4-2t}{3} = 1$

3. $\frac{2x+1}{3} - \frac{x-5}{4} = \frac{1}{2}$

4. $\frac{3m+2}{5m} + \frac{2m-1}{2m} = 4$

5. $\frac{4}{x-1} = \frac{x+1}{12}$

6. $\frac{x}{x-2} + \frac{4}{x-2} = 10$

7. **NAVIGATION** The current in a river is 6 miles per hour. In her motorboat Marissa can travel 12 miles upstream or 16 miles downstream in the same amount of time. What is the speed of her motorboat in still water?

8. **WORK** Adam, Bethany, and Carlos own a painting company. To paint a particular house alone, Adam estimates that it would take him 4 days, Bethany estimates $5\frac{1}{2}$ days, and Carlos 6 days. If these estimates are accurate, how long should it take the three of them to paint the house if they work together?

NAME _____ DATE _____ PERIOD ____

9-6 Study Guide and Intervention *(continued)*

Solving Rational Equations and Inequalities

Solve Rational Inequalities To solve a rational inequality, complete the following steps.

Step 1	State the excluded values.
Step 2	Solve the related equation.
Step 3	Use the values from steps 1 and 2 to divide the number line into regions. Test a value in each region to see which regions satisfy the original inequality.

Example **Solve** $\frac{2}{3n} + \frac{4}{5n} \leq \frac{2}{3}$.

Step 1 The value of 0 is excluded since this value would result in a denominator of 0.

Step 2 Solve the related equation.

$$\frac{2}{3n} + \frac{4}{5n} = \frac{2}{3}$$ Related equation

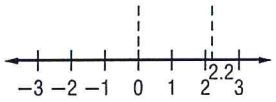
$$15n\left(\frac{2}{3n} + \frac{4}{5n}\right) = 15n\left(\frac{2}{3}\right)$$ Multiply each side by $15n$.

$$10 + 12 = 10n$$ Simplify.

$$22 = 10n$$ Simplify.

$$2.2 = n$$ Simplify.

Step 3 Draw a number with vertical lines at the excluded value and the solution to the equation.



Test $n = -1$. $-\frac{2}{3} + \left(-\frac{4}{5}\right) \leq \frac{2}{3}$ is true.

Test $n = 1$. $\frac{2}{3} + \frac{4}{5} \leq \frac{2}{3}$ is *not* true.

Test $n = 3$. $\frac{2}{9} + \frac{4}{15} \leq \frac{2}{3}$ is true.

The solution is $n < 0$ or $n \geq 2.2$.

Exercises

Solve each inequality.

1. $\frac{3}{a+1} \geq 3$

2. $\frac{1}{x} \geq 4x$

3. $\frac{1}{2p} + \frac{4}{5p} > \frac{2}{3}$

4. $\frac{3}{2x} - \frac{2}{x} > \frac{1}{4}$

5. $\frac{4}{x-1} + \frac{5}{x} < 2$

6. $\frac{3}{x^2-1} + 1 > \frac{2}{x-1}$

NAME _____ DATE _____ PERIOD ____

9-6 Study Guide and Intervention

Solving Rational Equations and Inequalities

Solve Rational Equations A **rational equation** contains one or more rational expressions. To solve a rational equation, first multiply each side by the least common denominator of all of the denominators. Be sure to exclude any solution that would produce a denominator of zero.

Example Solve $\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$.

$\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$ Original equation

$10(x+1)\left(\frac{9}{10} + \frac{2}{x+1}\right) = 10(x+1)\left(\frac{2}{5}\right)$ Multiply each side by $10(x+1)$.

$9(x+1) + 2(10) = 4(x+1)$ Multiply.

$9x + 9 + 20 = 4x + 4$ Distributive Property

$5x = -25$ Subtract $4x$ and 29 from each side.

$x = -5$ Divide each side by 5.

Check

$\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$ Original equation

$\frac{9}{10} + \frac{2}{-5+1} \stackrel{?}{=} \frac{2}{5}$ $x = -5$

$\frac{9}{10} + \frac{2}{-4} \stackrel{?}{=} \frac{2}{5}$ Simplify.

$\frac{18}{20} - \frac{10}{20} \stackrel{?}{=} \frac{2}{5}$ Simplify.

$\frac{8}{20} \stackrel{?}{=} \frac{2}{5}$ Simplify.

$\frac{2}{5} = \frac{2}{5}$

Exercises

Solve each equation.

1. $\frac{2y}{3} - \frac{y+3}{6} = 2$ 5
2. $\frac{4t-3}{5} - \frac{4-2t}{3} = 1$ 2
3. $\frac{2x+1}{3} - \frac{x-5}{4} = \frac{1}{2}$ $-\frac{13}{5}$
4. $\frac{3m+2}{5m} + \frac{2m-1}{2m} = 4$ $-\frac{1}{24}$
5. $\frac{4}{x-1} = \frac{x+1}{12}$ ± 7
6. $\frac{x}{x-2} + \frac{4}{x-2} = 10$ $\frac{8}{3}$

7. **NAVIGATION** The current in a river is 6 miles per hour. In her motorboat Marissa can travel 12 miles upstream or 16 miles downstream in the same amount of time. What is the speed of her motorboat in still water? 42 mph

8. **WORK** Adam, Bethany, and Carlos own a painting company. To paint a particular house alone, Adam estimates that it would take him 4 days, Bethany estimates $5\frac{1}{2}$ days, and Carlos 6 days. If these estimates are accurate, how long should it take the three of them to paint the house if they work together? about $1\frac{2}{3}$ days

Lesson 9-6

NAME _____ DATE _____ PERIOD ____

9-6 Study Guide and Intervention *(continued)*

Solving Rational Equations and Inequalities

Solve Rational Inequalities To solve a rational inequality, complete the following steps.

Step 1 State the excluded values.
Step 2 Solve the related equation.
Step 3 Use the values from steps 1 and 2 to divide the number line into regions. Test a value in each region to see which regions satisfy the original inequality.

Example Solve $\frac{2}{3n} + \frac{4}{5n} \leq \frac{2}{3}$.

Step 1 The value of 0 is excluded since this value would result in a denominator of 0.

Step 2 Solve the related equation.

$\frac{2}{3n} + \frac{4}{5n} = \frac{2}{3}$ Related equation

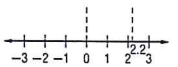
$15n\left(\frac{2}{3n} + \frac{4}{5n}\right) = 15n\left(\frac{2}{3}\right)$ Multiply each side by $15n$.

$10 + 12 = 10n$ Simplify.

$22 = 10n$ Simplify.

$2.2 = n$ Simplify.

Step 3 Draw a number with vertical lines at the excluded value and the solution to the equation.



Test $n = -1$. $-\frac{2}{3} + \left(-\frac{4}{5}\right) \leq \frac{2}{3}$ is true.

Test $n = 1$. $\frac{2}{3} + \frac{4}{5} \leq \frac{2}{3}$ is *not* true.

Test $n = 3$. $\frac{2}{9} + \frac{4}{15} \leq \frac{2}{3}$ is true.

The solution is $n < 0$ or $n \geq 2.2$.

Exercises

Solve each inequality.

1. $\frac{3}{a+1} \geq 3$ $-1 < a \leq 0$
2. $\frac{1}{x} \geq 4x$ $x \leq -\frac{1}{2}$ or $0 < x \leq \frac{1}{2}$
3. $\frac{1}{2p} + \frac{4}{5p} > \frac{2}{3}$ $0 < p < \frac{39}{20}$
4. $\frac{3}{2x} - \frac{2}{x} > \frac{1}{4}$ $-2 < x < 0$
5. $\frac{4}{x-1} + \frac{5}{x} < 2$ $x < 0$ or $\frac{1}{2} < x < 1$ or $x > 5$
6. $\frac{3}{x^2-1} + 1 > \frac{2}{x-1}$ $x < -1$ or $0 < x < 1$ or $x > 2$

Answers

NAME _____ DATE _____ PERIOD ____

9-6 Practice

Solving Rational Equations and Inequalities

Solve each equation or inequality. Check your solutions.

1. $\frac{12}{x} + \frac{3}{4} = \frac{3}{2}$

2. $\frac{x}{x-1} - 1 = \frac{x}{2}$

3. $\frac{p+10}{p^2-2} = \frac{4}{p}$

4. $\frac{s}{s+2} + s = \frac{5s+8}{s+2}$

5. $\frac{5}{y-5} = \frac{y}{y-5} - 1$

6. $\frac{1}{3x-2} + \frac{5}{x} = 0$

7. $\frac{5}{t} < \frac{9}{2t+1}$

8. $\frac{1}{2h} + \frac{5}{h} = \frac{3}{h-1}$

9. $\frac{4}{w-2} = \frac{-1}{w+3}$

10. $5 - \frac{3}{a} < \frac{7}{a}$

11. $\frac{4}{5x} + \frac{1}{10} < \frac{3}{2x}$

12. $8 + \frac{3}{y} > \frac{19}{y}$

13. $\frac{4}{p} + \frac{1}{3p} < \frac{1}{5}$

14. $\frac{6}{x-1} = \frac{4}{x-2} + \frac{2}{x+1}$

15. $g + \frac{g}{g-2} = \frac{2}{g-2}$

16. $b + \frac{2b}{b-1} = 1 - \frac{b-3}{b-1}$

17. $2 = \frac{x+2}{x-3} + \frac{x-2}{x-6}$

18. $5 - \frac{3d+2}{d-1} = \frac{2d-4}{d+2}$

19. $\frac{1}{n+2} + \frac{1}{n-2} = \frac{3}{n^2-4}$

20. $\frac{c+1}{c-3} = 4 - \frac{12}{c^2-2c-3}$

21. $\frac{3}{k-3} + \frac{4}{k-4} = \frac{25}{k^2-7k+12}$

22. $\frac{4v}{v-1} - \frac{5v}{v-2} = \frac{2}{v^2-3v+2}$

23. $\frac{y}{y+2} + \frac{7}{y-5} = \frac{14}{y^2-3y-10}$

24. $\frac{x^2+4}{x^2-4} + \frac{x}{2-x} = \frac{2}{x+2}$

25. $\frac{r}{r+4} + \frac{4}{r-4} = \frac{r^2+16}{r^2-16}$

26. $3 = \frac{6a-1}{2a+7} + \frac{22}{a+5}$

27. **BASKETBALL** Kiana has made 9 of 19 free throws so far this season. Her goal is to make 60% of her free throws. If Kiana makes her next x free throws in a row, the function $f(x) = \frac{9+x}{19+x}$ represents Kiana's new ratio of free throws made. How many successful free throws in a row will raise Kiana's percent made to 60%?

28. **OPTICS** The lens equation $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ relates the distance p of an object from a lens, the distance q of the image of the object from the lens, and the focal length f of the lens. What is the distance of an object from a lens if the image of the object is 5 centimeters from the lens and the focal length of the lens is 4 centimeters?

NAME _____ DATE _____ PERIOD ____

9-6 Skills Practice

Solving Rational Equations and Inequalities

Solve each equation or inequality. Check your solutions.

1. $\frac{x}{x-1} = \frac{1}{2}$ -1

2. $2 = \frac{4}{n} + \frac{1}{3}$ $\frac{12}{5}$

3. $\frac{9}{3x} = \frac{-6}{2}$ -1

4. $3 - z = \frac{2}{z}$ $1, 2$

5. $\frac{2}{d+1} = \frac{1}{d-2}$ 5

6. $\frac{s-3}{5} = \frac{8}{s}$ $-5, 8$

7. $\frac{2x+3}{x+1} = \frac{3}{2}$ -3

8. $-\frac{12}{y} = y - 7$ $3, 4$

9. $\frac{x-2}{x+4} = \frac{x+1}{x+10}$ 8

10. $\frac{3}{k} - \frac{4}{3k} > 0$ $k > 0$

11. $2 - \frac{3}{v} < \frac{5}{v}$ $0 < v < 4$

12. $n + \frac{3}{n} < \frac{12}{n}$ $n < -3$ or $0 < n < 3$

13. $\frac{1}{2m} - \frac{3}{m} < -\frac{5}{2}$ $0 < m < 1$

14. $\frac{1}{2x} < \frac{2}{x} - 1$ $0 < x < \frac{3}{2}$

15. $\frac{15}{x} + \frac{9x-7}{x+2} = 9$ 3

16. $\frac{3b-2}{b+1} = 4 - \frac{b+2}{b-1}$ 4

17. $2 = \frac{5}{2q} + \frac{2q}{q+1}$ -5

18. $8 - \frac{4}{z} = \frac{8z-8}{z+2}$ $\frac{2}{5}$

19. $\frac{1}{n+3} + \frac{5}{n^2-9} = \frac{2}{n-3}$ -4

20. $\frac{1}{w+2} + \frac{1}{w-2} = \frac{4}{w^2-4}$ $\varnothing$

21. $\frac{x-8}{2x+2} + \frac{x}{2x-2} = \frac{2x-3}{x+1}$ $\varnothing$

22. $\frac{12s+19}{s^2+7s+12} - \frac{3}{s+3} = \frac{5}{s+4}$ 2

23. $\frac{2e}{e^2-4} + \frac{1}{e-2} = \frac{2}{e-2}$ -6

24. $\frac{8}{t^2-9} + \frac{4}{t-3} = \frac{2}{t-3}$ 5

NAME _____ DATE _____ PERIOD ____

9-6 Practice (Average)

Solving Rational Equations and Inequalities

Solve each equation or inequality. Check your solutions.

1. $\frac{12}{x} + \frac{3}{4} = \frac{3}{2}$ 16

2. $\frac{x}{x-1} - 1 = \frac{x}{2}$ $-1, 2$

3. $\frac{p+10}{p^2-2} = \frac{4}{p}$ $-\frac{2}{3}, 4$

4. $\frac{s}{s+2} + s = \frac{5s+8}{s+2}$ 4

5. $\frac{5}{y-5} = \frac{y}{y-5} - 1$ all reals except 5

6. $\frac{1}{3x-2} + \frac{5}{x} = 0$ $\frac{5}{8}$

7. $\frac{5}{t} < \frac{9}{2t+1}$ $t < -5$ or $-\frac{1}{2} < t < 0$

8. $\frac{1}{2h} + \frac{5}{h} = \frac{3}{h-1}$ $\frac{11}{5}$

9. $\frac{4}{w-2} = \frac{-1}{w+3}$ -2

10. $5 - \frac{3}{a} < \frac{7}{a}$ $0 < a < 2$

11. $\frac{4}{5x} + \frac{1}{10} < \frac{3}{2x}$ $0 < x < 7$

12. $8 + \frac{3}{y} > \frac{19}{y}$ $y < 0$ or $y > 2$

13. $\frac{4}{p} + \frac{1}{3p} < \frac{1}{5}$ $p < 0$ or $p > \frac{65}{3}$

14. $\frac{6}{x-1} = \frac{4}{x-2} + \frac{2}{x+1}$ $\varnothing$

15. $g + \frac{g}{g-2} = \frac{2}{g-2}$ -1

16. $b + \frac{2b}{b-1} = 1 - \frac{b-3}{b-1}$ -2

17. $2 = \frac{x+2}{x-3} + \frac{x-2}{x-6}$ $\frac{14}{3}$

18. $5 - \frac{3d+2}{d-1} = \frac{2d-4}{d+2}$ 6

19. $\frac{1}{n+2} + \frac{1}{n-2} = \frac{3}{n^2-4}$ $\frac{3}{2}$

20. $\frac{c+1}{c-3} = 4 - \frac{12}{c^2-2c-3}$ $-\frac{5}{3}, 5$

21. $\frac{3}{k-3} + \frac{4}{k-4} = \frac{25}{k^2-7k+12}$ 7

22. $\frac{4v}{v-1} - \frac{5v}{v-2} = \frac{2}{v^2-3v+2}$ $-1, -2$

23. $\frac{y}{y+2} + \frac{7}{y-5} = \frac{14}{y^2-3y-10}$ 0

24. $\frac{x^2+4}{x^2-4} + \frac{x}{2-x} = \frac{2}{x+2}$ $\varnothing$

25. $\frac{r}{r-4} + \frac{4}{r-4} = \frac{r^2+16}{r^2-16}$ all reals except -4 and 4

26. $3 = \frac{6a-1}{2a+7} + \frac{22}{a+5}$ -2

27. **BASKETBALL** Kiana has made 9 of 19 free throws so far this season. Her goal is to make 60% of her free throws. If Kiana makes her next x free throws in a row, the function $f(x) = \frac{9+x}{19+x}$ represents Kiana's new ratio of free throws made. How many successful free throws in a row will raise Kiana's percent made to 60%? 6

28. **OPTICS** The lens equation $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ relates the distance p of an object from a lens, the distance q of the image of the object from the lens, and the focal length f of the lens. What is the distance of an object from a lens if the image of the object is 5 centimeters from the lens and the focal length of the lens is 4 centimeters? 20 cm